Elemente de combinatorică

1. Să se rezolve:
a) $A_x^5 - 3A_x^4 = 21A_x^3$
b) $8C_{x+2}^5 = 3A_{x+1}^3$
c) $A_x^{10} + A_x^9 = 9A_x^8$
d) $4C_{x-1}^4 = 15A_{x-1}^3$
e) $P_{n+1} = 8(P_n + P_{n-1})$
f) A_n^2=72
g) $A_x^5 - 3A_x^4 = 21A_x^3$
h) $8C_{x+2}^5 = 3A_{x+1}^3$
i) $(n+3)P_{n+1} = P_n + P_{n+2}$
j) C_n^2=21
k) $A_x^{10} + A_x^9 = 9A_x^8$

2. Să se rezolve:
a) $\begin{cases} A_{2y}^{3x} = 8A_{2y}^{3x-1} \\ 9C_{2y}^{3x} = 8C_{2y}^{3x-1} \end{cases}$
b) $\begin{cases} A_x^y = 7A_x^{y-1} \\ 6C_x^y = 5C_x^{y+1} \end{cases}$
c) $\begin{cases} 4C_x^{y+1} = 5C_x^y \\ 3C_{x-1}^y = 5C_{x-1}^{y-1} \end{cases}$
d) $C_{x+1}^6 < C_{x+1}^4$
e) $C_{13}^x < C_{13}^{x+2}$
f) $C_x^6 < C_x^4$
g) $C_{13}^{x-1} < C_{13}^{x+1}$

3. Să se calculeze:
a) $S = \frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \cdots + \frac{n+2}{n!+(n+1)!+(n+2)!}$
b) $S = 3 \cdot 1! + 7 \cdot 2! + \cdots + (n^2 + n + 1) \cdot n!$
c) $S = \frac{1}{2!} + \frac{5}{3!} + \cdots + \frac{n^2-n-1}{n!}$
d) $S = \frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \cdots + \frac{n+2}{n!+(n+1)!+(n+2)!}$
e) $S = 1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n!$

4. Să se arate că:
   a) $C_7^7 + C_8^7 + C_9^7 + \cdots + C_{20}^7 = C_{21}^8$
   b) $C_{10}^{10} + C_{11}^{10} + C_{12}^{10} + \cdots + C_{30}^{10} = C_{31}^{11}$
   c) $C_2^2 + C_3^2 + \cdots + C_n^2 = C_{n+1}^3$

5. Să se calculeze:
   a) $S = C_n^1 + 2C_n^2 + 3C_n^3 + \cdots + nC_n^n$
   b) $S = C_n^0 + \frac{1}{2}C_n^1 + \frac{1}{3}C_n^2 + \cdots + \frac{1}{n+1}C_n^n$
   c) $S = C_n^0 + 3C_n^1 + 5C_n^2 + \cdots + (2n+1)C_n^n$
   d) $S = C_n^0 + 4C_n^1 + 7C_n^2 + \cdots + (3n+1)C_n^n$